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**DOUBLE-CIRCULAR ARC HELICAL GEARS:  
MAIN FEATURES, BRIEF HISTORY OF DEVELOPMENT,  
INDUSTRIAL APPLICATION, COMPARISON WITH DOUBLE-HELICAL  
INVOLUTE GEARS AND CONCLUSION: PERSPECTIVES FOR  
INDUSTRIAL APPLICATION IN THE USA**

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**1. Main Features**

The manufacturing of double-circular arc helical gears is based on application of two hobs of the same pitch. The normal section of each hob represents an imaginary rack-cutter whose shape is piecewise Fig 1 shows the shape of the imaginary rack-cutter that has been standardized throughout many countries in ASIA. Fig. 2 shows the shapes of two imaginary rack-cutters. The dimensions of two hobs that are designed in accordance with these standards depend only on the pitch, of to-be generated helical gears and do not depend on the helix angle and numbers of teeth of the helical gears.

The tooth surfaces of mating double-circular arc helical gears contact each other at every instant at two points that are located up and down toward the pitch plane. The contact points move in the process of meshing along two straight lines that are located in the fixed coordinate system rigidly connected to the gear housing. These two straight lines represent the so-called lines of action. One of these lines lies in a plane that is located up to the pitch plane and the other one lies in a plane that is located down toward the pitch plane (Fig 3 and Fig. 4). The lines of action are collinear to the gear axes.

Due to the elasticity of gear tooth surfaces the real contact under the load is spread over an ellipse at every instant. The center of this ellipse moves in the fixed coordinate system along

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the line of action.

Double circular-arc helical gears cause an axial load on the bearings. To eliminate this load two pairs of gears with opposite directions of the helices for both pinions of the two pairs of gears drives are used. This design is the same as it was used for regular involute gears.

## 2. Brief History of Development

Until 1954 only involute helical gears have been used in the industry. In the year of 1954 Novikov [8] has proposed helical gears with singular arc teeth in the cross-section that is perpendicular to the gear axis. The main feature of his invention was the mismatch of radii of two circular arcs that represent the cross-sections of the mating gears. Such gears could provide a point contact at every instant. Only one line of action could be provided instead of two lines provided by double-circular arc teeth.

Initially circular arc teeth have been proposed by Wildhaber in USA [10]. This is the reason why singular arc helical teeth are now called Wildhaber-Novikov helical gears. However, there is a great difference between the two types of helical gears that have been proposed by Wildhaber and Novikov.

Wildhaber's gears are in line contact but not in point contact at every instant. Their shapes in a cross-section that is perpendicular to gear axes are conjugate and a mismatch cannot be provided.

Novikov helical gears in point contact at every instant. Their shapes in cross-section are not conjugated and a mismatch can be provided. Due to the small difference of radii of curvature and since a concave and convex shapes are in tangency, the contact stresses in comparison with involute helical gears may be substantially reduced.

The development of helical gears with double-circular arc teeth was a substantial step forward. This development included progressive ideas that have been proposed by a group of

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researchers:

- (1) Generation of gears by two hobs whose normal sections represent circular arcs (proposed by H. Winter and Looman [11] and V. Kudrjavitzev, USSR [3]). Litvin [4] and Davidov [2] have developed the method for generation of helical gears by two rigidly connected imaginary rack-cutters. Taking into account the above mentioned method for generation we have to say that the term "circular arc teeth" should be considered as a conventional term only. In reality the shapes of modified Wildhaber-Novikov gears differ from a circular arc.
- (2) The second advantage of double circular arc teeth is existence of two lines of action that could increase the resistance of gears to bending stresses.

3. Industrial Application

It is without doubts that double circular arc helical gears will find and have found already an industrial application. There is a certain area where such gears are competitive with involute helical gears, i.e. the area where the hardness is less HB 350, the gears are not ground and the gear velocity on the pitch circles is less than 60 ft./sec. A typical example of such gears are the reducers of Darco USA Pumping Units.

Double circular arc helical gears are successfully used throughout ASIA. There is an example of application of Wildhaber-Novikov gears in the Western World - in England - by Westland Helicopter Company. It is surprising that Westland has designed and produced helical gears with single circular arc teeth but not with double arc teeth.

The perspectives of further application of helical gears with double circular arc teeth should be considered by comparison of their advantages and disadvantages with respect to regular involute helical gears.

4. Comparison of Double Circular Arc Helical Gears with Helical Involute Gears

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The analysis of competitive application of helical gears with double circular arc teeth and involute helical gears should be done on the basis of following criteria:

(i) **Contact Stresses**

Contact stresses of double circular arc helical gears are approximately in two times less than for involute helical gears with the hardness  $H \leq 350$  HB (based on information adopted from the book "Machine Elements" by V.N. Kudrjatzev, 1980, in Russian, USSR).

(ii) **Bending Stresses**

The information is still contradictive. Although the resistance to bending stresses of double circular teeth has been increased in comparison with single circular arc teeth, the bending stresses are still larger than the bending stresses of involute helical gears.

(iii) **Adjustment by Lapping**

The adjustment of gears by lapping is much better for double circular arc helical gears than for involute helical gears. This is the result of contact of convex-concave shapes by double circular helical gears.

(iv) **Technological Advantages and Disadvantages**

(a) **Double Circular Arc Teeth**: It is almost impossible to grind the gears, it is necessary to use two hobs for the given pitch, the shape of the hobs is more complicated one. However, the production of hobs is simple enough and can be definitely performed by American tool companies. The production of those hobs may be compared with production of hobs for chain gears and multi-dowelled shafts.

(b) **Involute Helical Gears**. It is easy to grind the gears, it is possible to use only one hob for the given pitch, the shape of the hob is very simple (it is a straight line). These gears are more

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perspective when a high velocity and high precision are required.

5. Sensitivity to Misalignment, Level of Noise

Helical gears with double circular arc teeth are more sensitive to the change of center distance than involute helical gears. However, involute helical gears are more sensitive than double circular arc helical gears to the axis misalignment. It is necessary to crown the pinion of involute helical gears to localize the bearing contact. Due to the specific geometry of double circular arc helical gears (the surfaces are in point contact) their bearing contact is localized already and crowning is not required.

The level of transmission errors and the resulted noise is higher for double circular arc helical gears than for involute helical gears. This is the result of the lower precision of the tool due to the more complicated tool shape.

6. Conclusion: Perspectives for Industrial Application in the USA

The industrial application of helical gears with double circular arc teeth in the USA should be considered as one of the needs in the gear industry. The areas of application of such gears are the machines of petroleum industry, machines of mining industry, rolling mills etc. The application of such gears instead of involute helical gears allows to avoid grinding, application of alloyed steel. It is not reasonable to apply double crossed helical gears where the velocity is larger than 60 ft./sec. It do not foresee difficulties in the production of the tools. I propose: (i) to develop a National Standard for double-circular arc helical gears. (b) start experimental production and test of above mentioned gears.

I believe that such research projects can be developed under the supervision of American Petroleum Institute. I also recommend to consider helical gears with double circular arc teeth as

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an excellent application for oil pumping units.

\* Dr. Litvin is considered one of the foremost gear experts in the world and currently is the Chairman of the ASME subcommittee on gear geometry.

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