Introduction for Development, Application and Performance of Double Circular Arc Gear

Double Circular Arc (DCA) gear is researched since 1965 in China. After many years investigation, research and test in the gear research rooms in several university and gear research institutions, the Tooth Form Standard for DCA gear was determined in 1981 and the DCA gear are used widely in Chinese metallurgy and mine machinery, textile machinery and oil producing equipment since then. China is a large oil production country and currently it has more than tens thousands of pumping units under use in various oilfields in China. All of the gear reducers used on these pumping units are DCA gear reducers and involute gear had been eliminated by the end of the 1970s. The reason of elimination of involute gear is that overall pittings appeared on this kind of involute gear which were used on the pumping units. The main advantages of DCA gear are the higher contact strength than involute gear's and stronger in pitting-resistance. The bending strength of DCA gear is a slight higher than involute gear's. So the DCA gear are commonly welcomed by many customers in the world.

Up till now, LS Brand pumping unit with DCA gear reducers have been exported to more than ten countries, such as: USA, Canada, Argentina, Venezuela, Brazil, Ecuador, Egypt, Oman, India, Indonesia, Azerbaijan, Congo and Gabon, etc.

Introduction for Double Circular Arc Gear Design and Calculation

1. Type, feature and application for circular arc gear drive

Circular arc gear drive is a new kind of gear drive developed in recent tens years. Since 1958, Our country has had large quantity of research, test and popularization in factories, universities and scientific research institutions. And currently it has been used widely in metallurgy, mine, lifting & transportation machinery as well as high speed gear drive.

Fig. 1 is the outside review for circular arc gear drive. It's a kind of helical (or herringbone) gear taking circular arc as the tooth form. In order to convenient machining, we usually make the normal plane tooth form as circular arc and transverse profile as approximate circular arc. According to the tooth form of circular arc gear, the circular arc gear is divided into single circular arc gear drive and double circular arc (DCA) gear drive. We introduce mainly DCA gear drive herewith. As show in Fig.2 for the DCA gear drive, the large and small gears adopt same tooth profile, the tooth profile at part of addendum is convex circular arc and the tooth profile at the part of addendum is concave circular arc, the whole tooth profile consists of convex and concave circular arcs.

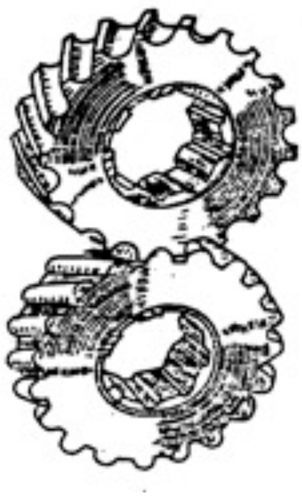

Fig. 1

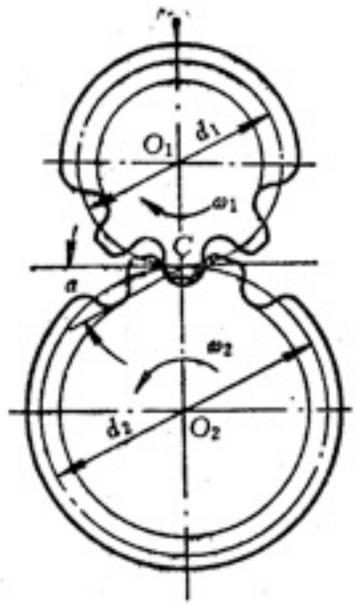

Fig.2 DCA gear drive

1.1 DCA gear drive

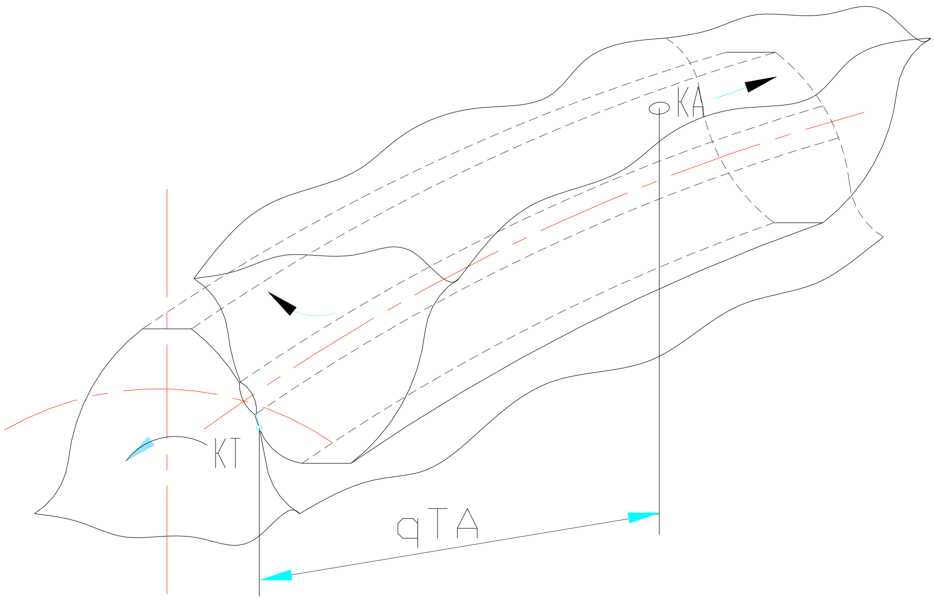

Fig. 3 Sketch of engagement of DCA gear drive

As show in Fig. 3, the large and small gears of DCA gear drive adopt same tooth form and their tooth forms consist of two sections of circular arcs, its addendum part is convex circular arc and dedendum part is concave circular arc. So, the DCA gear drive is equalent to the two pairs of single circular arc gears to be compounded to work. In the course of drive, one pair is driven by convex tooth to concave tooth and instantaneous contact point K_T; and the other pair is driven by concave tooth to convex tooth and instantaneous contact point K_A. So, during drive, at front and rear of pitch point, there are two contacting lines at same time and the instantaneous contact points K_T and K_A which will move axially along with their self contacting lines. These two instantaneous contact points K_T and K_A are located in two different end sections, the axial-distance q_{TA} is called axial distance of both simultaneous contact points on same tooth. It's because one pair of tooth surface has two points to be contacted simultaneously at two contacting lines, this kind of drive is called double contacting lines drive.

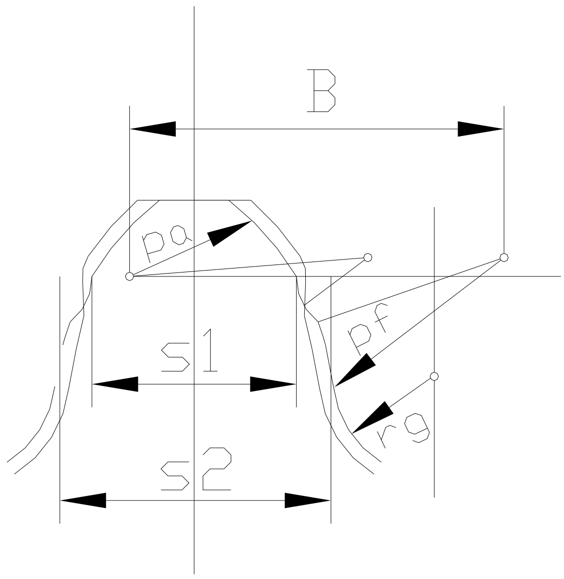

Fig. 4 Basic tooth form of step type DCA gear

Fig.4 is the basic tooth form of step type DCA gear, the thickness of its addendum part (convex tooth) is reduced, and the thickness of dedendum part (concave tooth) increased. So the un-working tooth surface between convex and concave tooth forms formed a step, the transition curve in this area is a little arc. At the time of engagent, this kind of gear, in the un-working tooth surfaces, will form a larger space to avoide the defects on un-working tooth surfaces contact. In addition, because of increase of thickness at dedendum so that the bending strength at dedendum is increased comparing with common tangent type circular arc gear and also if the ratio of pitch thickness S_2/S_1 is selected properly, the bending strengthes in the waist of tooth and dedendum are approximate equal. Thus obtain max. bearing capability. The bearing capability of step type DCA gear is about 40~60% higher than single circular arc gear. Since step type DCA gear drive has a serial advantages and received widely attention in the line of gear in different countries, currently it's under gradually popularization and application.

DCA gear drive has the following features:

1) High bending strength. Under the condition of same geometric parametas, the simultaneous working contact points increase one time and accordingly, the load shared on each contact point will be half in the theory, so the strength of DCA gear is higher. The bending strength shall be 30% higher than involute gear if its tooth form design is appropriate.

2) High contact strength. In addition to more contact points, the total length of two transient contact lines formed after running is longer than single circular arc gear's and generally its pressure angle is selected smaller than single circular are gear's, so the contact strength of DCA gear is obviously higher than SCA gear's.
3) Both gears of DCA gear drive adopt convex teeth for addendum and concave teeth for dedendum: convex-concave teeth form, so it could use one hobbing cutter for one pair of gear cutting.
4) More stable drive, less vibration and noise.

2. Engagement feature of circular arc gear drive

The engagement feature of DCA gear drive is important quality index to check the stability of gear drive. In order to guarantee the gear drive stably and continuously, it's not only to request the teeth surfaces of one pair gear to realize fixed drive ratio to drive but also request each pair of teeth "contact" stably, this need the coincidence degree to guarantee. Reasonable selection of coincidence degree is not only to guarantee the drive stability but also to increase the bearing capability of drive, especially in DCA gear drive.

2.2 Engagement feature of DCA gear drive

2.2.1 Axial-distance q_{TA} between two simultaneously contact points on the same working teeth surfaces.

According to law that the common tangent of both teeth surfaces at contact points must be crossed with pitch line, we may calculate approximately the axial-distance q_{TA} between both the simultaneous contacted points k_T and k_A on the working teeth surfaces, as per to Fig.5:

$$q_{TA}=\frac{0.5\pi m_n+2l_\alpha-0.5j_n}{\sin\beta} - 2\rho_\alpha \cos\alpha_n \sin\beta \qquad (1)$$

In which: j_n is normal side tolerance.

Ratio of q_{TA} and axial tooth distance P_x is double points distance coefficient λ.

$$\lambda=\frac{q_{TA}}{P_x} \qquad (2)$$

The λ is defined not only by tooth form parameters, it will be changed as per the change of spiral angle β

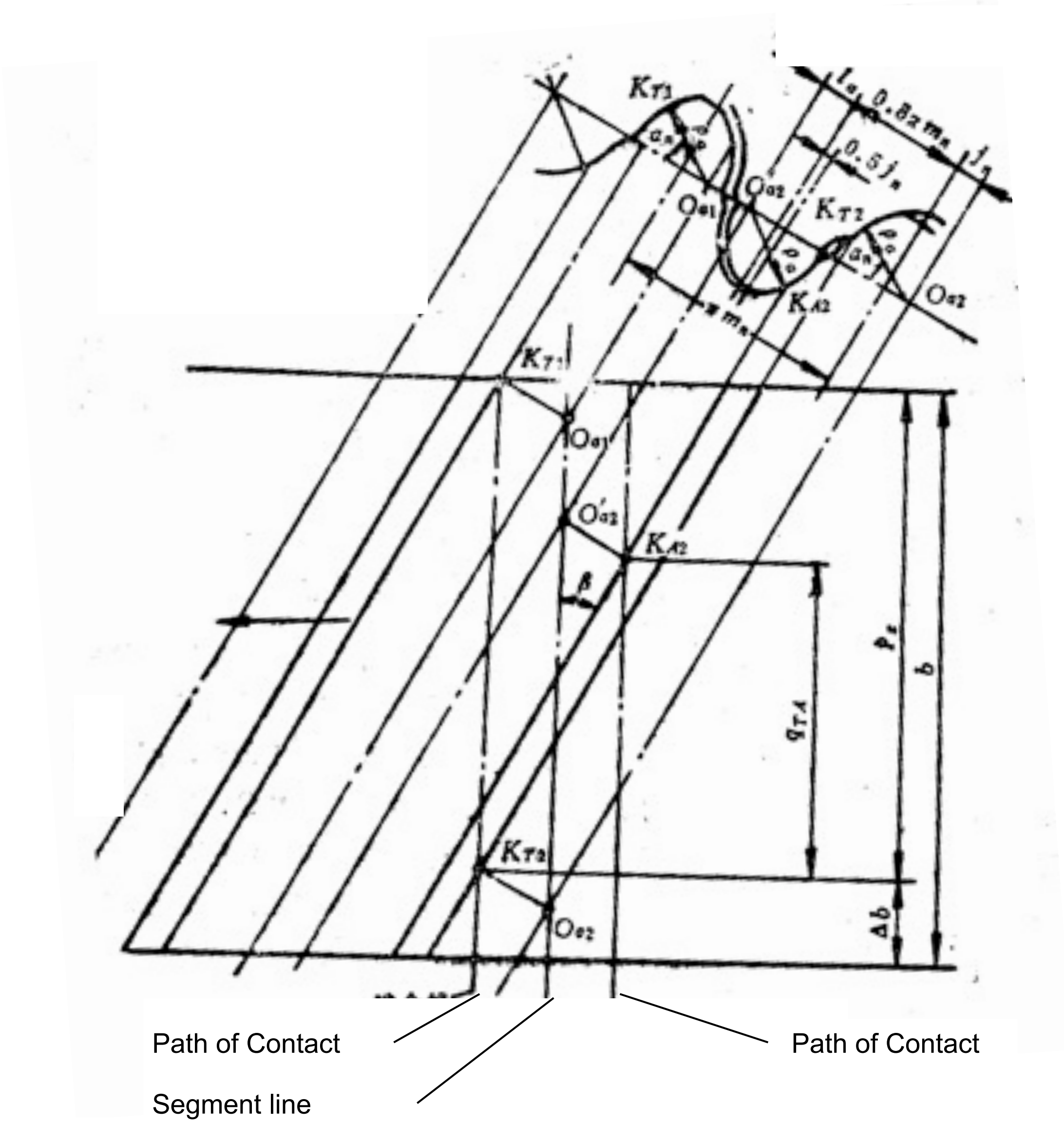

Fig. 5 Engagement characteristic for DCA gear (pitch circle developed view)

2.2.2 Multi points engagement coefficient

During the course of gear drive, the number of simultaneous contat point of teeth will be changed periodically. If the working width of gear $b=mp_x+\triangle b$ (m is integer, $\triangle b$ is mantissa) in the scope of turning one tooth, it may have $2m$ points, $2m+1$ points and $2m+2$ points contact, when the relevant contacting points working, the ratio of turned pitch circle arc length and circular pitch is called multi-point engagement coefficient and to be separalely as ε_{2md}, $\varepsilon_{(2m+1)d}$, $\varepsilon_{(2m+2)d}$. we will treat them as 3 kinds of situation according to the large or small of $\triangle b$ and q_{TA}, to calculate as per Table. 1.

For example: at the situation of showing in Fig. 5, $\triangle_b<(P_x-q_{TA})$, so,

$$\varepsilon_{2d}=1-\frac{2\triangle b}{P_x}, \qquad \varepsilon_{3d}=2\frac{\triangle b}{P_x}.$$

Table 1 Calculation equation for multi-point engagement coefficient

Name of engagement coefficient	Code	Situation I	Situation II	Situation III
		When $\triangle b \le P_x-q_{TA}$	When $(P_x-q_{TA}) \le \triangle b \le q_{TA}$	When $\triangle b \ge q_{TA}$
2m points engagement coefficient	ε_{2md}	$1-\frac{2\triangle b}{P_x}$	$\frac{q_{TA}-\triangle b}{P_x}$	——
(2m+1) points engagement coefficient	$\varepsilon_{(2m+1)d}$	$\frac{2\triangle b}{P_x}$	$\frac{2(P_x - q_{TA})}{P_x}$	$2-\frac{2\triangle b}{Px}$
(2m+2) points engagement coefficient	$\varepsilon_{(2m+2)d}$	——	$\frac{\triangle b-(P_x - q_{TA})}{P_x}$	$\frac{2\triangle b}{P_x}-1$

2.2.3 Multi-pair of teeth engagement coefficient

During drive, the working teeth pair number at same time will be changed periodically also. In the scope of turning on tooth, may be, it has m pair of teeth, *(m+1)* pairs of teeth and *(m+2)* pair of teeth to join the work. When the relevant teeth working in pairs, the ratio of turned pitch arc length and circular pitch is called multi-pair teeth engagement coefficient. To be as ε_{mz},$\varepsilon_{(m+1)z}$ and $\varepsilon_{(m+2)z}$, we may treat them as 2 kinds of situations according to $\triangle b$*'s* large or small and calculate per Table 2.

According to the situations showed in Fig. 5, $\triangle b \le P_x-q_{TA}$, so, $\varepsilon_{1x}=1-\frac{q_{TA}+\triangle b}{P_x}$, $\varepsilon_{2x}=\frac{q_{TA}+\triangle b}{P_x}$. In which, the min. working teeth is one pair, so, when we calculate the strength, we should conside the condition of one pair of teeth and two points engagement.

Table 2 Calculation equation for multi-pair of teeth engagement coefficient

Name of engagement coefficient	Code	Situation I	Situation II
		When $\triangle b \leq (P_x - q_{TA})$	When $\triangle b \geq (P_x - q_{TA})$
Engagement coefficient for m pair of teeth	ε_{mz}	$1-\dfrac{q_{TA}+\triangle b}{P_x}$	——
Engagement coefficient for $(m+1)$ pair of teeth	$\varepsilon_{(m+1)z}$	$\dfrac{q_{TA}+\triangle b}{P_x}$	$2-\dfrac{q_{TA}+\triangle b}{P_x}$
Engagement coefficient for $(m+2)$ pair of teeth	$\varepsilon_{(m+2)z}$	——	$\dfrac{q_{TA}+\triangle b}{P_x}-1$

2.2.4 Determination of tooth width b

In the drive of double circular arc gear, it exists multi-pair of teeth engagement and multi-point engagement and the situation is complicated. So, if it requires different engaged teeth pairs and different contact point numbers, its min. tooth width b_{min}, is not same also. The min. tooth width b_{min} of DCA gear to be calculated per Table 3.

Table 3 Calculation Table of min. tooth width

Design requirements	Calculation equations
At least m pair of teeth and $2m$ of contact points work at same time.	$b_{min}=mP_x$
At least m pair of teeth and $2m-1$ of contact points work at same time.	$b_{min}=(m+\lambda-1)P_x$
At least m pair of teeth and $2m-2$ of contact points work at same time.	$b_{min}=(m-\lambda)\ P_x$

For example: at least 2 pairs of teeth and two points contact, the min. tooth width

$$b_{min}=(m-\lambda)P_x=(2-\lambda)P_x$$

at least 2 pairs of teeth and 3 points contact, the min. tooth width

$$b_{min}=(m+\lambda-1)P_x=(1+\lambda)P_x$$

The tooth width b to be determined as per following equation:

$$b=b_{min}+\triangle b_1 \qquad (3)$$

Selection of min. tooth width is recommended as per following equation:

$$b_{min}=(m-\lambda)\ P_x$$

$\triangle b$ to be selected as per following equation:

$$\triangle b=(0.15\sim0.35)\ P_x$$

3. DCA gear's basic tooth form and module series.

DCA gear's basic tooth form means basic rack's normal plane tooth form. For example: take tooth of basic rack as slot or the slot of basic rack as tooth, the tooth form formed from above is hobbing cutter's normal plane tooth form.

3.2 DCA gear's basic tooth form

In 1981, our country formulated the basic tooth form standard for DCA gear (JB2940-81), this standard is applicable to the DCA gear drive under the condition of tooth surface hardness not exceed to 350 HB and the tooth surface exceed to 350 HB without tooth surface grinding, normal plane module m_n=2~32mm. Please refer to Table 4 for basic tooth form and its parameters.

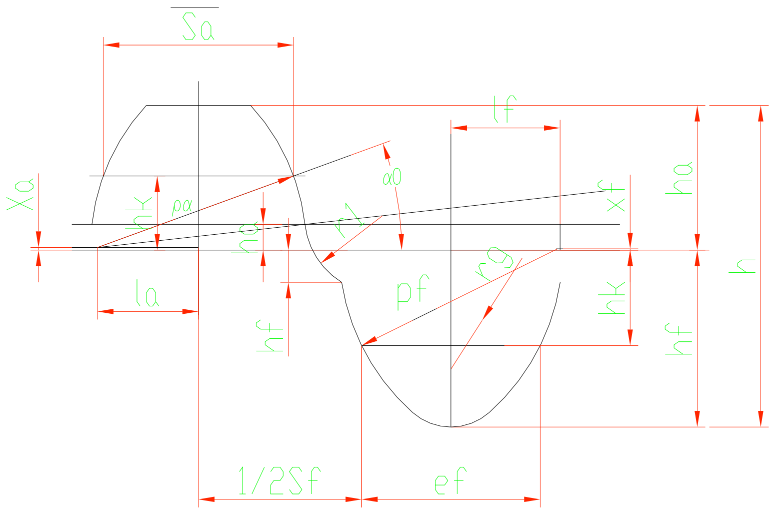

Fig. 6

Code: a_0—pressure angle; h—whole depth; h_a—addendum; h_f—inside pitch line length; ρ_a—circular arc radius of convex tooth flank profile; ρ_f—circular arc radius of concave tooth flank profile; x_a— travel motion of convex flank profile center;

x_f— travel motion of concave flank profile center; s_a—chordal thickness at convex tooth contact point; h_k—distance from contact point to pitch line; l_a—offset value of convex flank profile center; l_f—offset value of concave flank profile center; h_{ja}—distance from the tangent point connecting circular arc and concave tooth arc to pitch line; h_{jf}— distance from intersection point connecting arc and concave tooth arc to pitch line; e_f—tooth slot width at concave tooth contact point; s_f—chordal thickness at concave tooth contact point; δ_1—convex tooth processing angle; δ_2—concave tooth processing angle; r_j—connecting circular arc radius at tooth waist; r_g—circular arc radius at dedendum; j—side tolerance.

Table 4 Basic tooth form and its parameters

Normal module m_n mm	Parameters of basic tooth form										
	a_0	h^*	h^*_a	h^*_f	ρ^*_a	ρ^*_f	x^*_a	x^*_f	$\bar{s}^*_a$	h^*_K	l^*_a
2~3	24°	2	0.9	1.1	1.3	1.42	0.0163	0.0325	1.1173	0.5450	0.6289
>3~6	24°	2	0.9	1.1	1.3	1.41	0.0163	0.0285	1.1173	0.5450	0.6289
>6~10	24°	2	0.9	1.1	1.3	1.395	0.0163	0.0224	1.1173	0.5450	0.6289
>10~16	24°	2	0.9	1.1	1.3	1.38	0.0163	0.0163	1.1173	0.5450	0.6289
>16~32	24°	2	0.9	1.1	1.3	1.36	0.0163	0.0081	1.1173	0.5450	0.6289

Normal module m_n mm	Parameters of basic tooth shape									
	l^*_f	h^*_{ja}	h^*_{jf}	e^*_f	$\bar{s}^*_f$	δ_1	δ_2	r^*_j	r^*_g	j^*
2~3	0.7086	0.16	0.20	1.1773	1.9643	6°20’52”	9°6’7”	0.5103	0.4030	0.06
>3~6	0.6994	0.16	0.20	1.1773	1.9643	6°20’52”	9°19’30”	0.5078	0.4004	0.06
>6~10	0.6957	0.16	0.20	1.1573	1.9843	6°20’52”	9°10’21”	0.4906	0.3710	0.04
>10~16	0.6820	0.16	0.20	1.1573	1.9843	6°20’52”	9°9’49”	0.4885	0.3663	0.04
>16~32	0.6638	0.16	0.20	1.1573	1.9843	6°20’52”	9°48’11”	0.4858	0.3598	0.04

Note: The size parameters with * in the table indicate the ratio of this size and normal module m_n, and times normal module m_n with these ratios to obtain this size’s value, for example: $h^*\cdot m_n=h$, $\rho^*_a\cdot m_n=\rho_a$, ……etc.

3.3 Module series of circular arc gear

Please refer to Table 5 for the normal module series of circular arc gear.

Table 5 Module m_n series for circular arc gear (GB1840-89)

mm

First series	1.5 2 2.5 3 4 5 6 8 10 12 16 20 25 32 40 50
Second series	2.25 2.75 3.5 4.5 5.5 7 9 14 18 22 28 36 45

4. Geometrical size calculation for DCA gear drive

Table 6 Geometrical size calculation for DCA gear drive

Name	Code	Calculation equation	
		Small gear	Big gear
Center distance	a	$a=\frac{1}{2}m_t(z_1+z_2)=\frac{m_n(z_1+z_2)}{2\cos\beta}$ a should meet strength requirement, the a of gear reducer shall select standard value.	
Normal module	m_n	Determined as per the tooth strength calculation or selected as per section 5.1 of this chapter, should select standard value.	
Transverse module	m_t	$m_t=\frac{m_n}{\cos\beta}$	
Tooth number	Z	$z_1=\frac{z\Sigma}{1+u}$	$z_2=uz_1$
Spiral angle	β	$\cos\beta=\frac{m_n}{m_t}=\frac{m_n(z_1+z_2)}{2a}$	
Diameter of reference circle	d	$d_1=\frac{m_nz_1}{\cos\beta}$	$d_2=\frac{m_nz_2}{\cos\beta}$
Diameter of addendum circle	d_a	$d_{a1}=d_1+2h_a$	$d_{a2}=d_2+2h_a$
Diameter of dedendum circle	d_f	$d_{f1}=d_1-2h_f$	$d_{f2}=d_2-2h_f$
Axial tooth spacing	P_x	$Px=\frac{\pi m_n}{\sin\beta}$	
Tooth width (half tooth width if herringbone gear)	b	$b=b_{min}+\triangle b$ b_{min} please see Table 23.3-3	
Calculation for measuring size			
Nominal chordal depth	$\bar{h}$	$\bar{h}_1=m_n[h^*-r^*_{a1}(1-\cos\theta)]$	$\bar{h}_2=m_n[h^*-r^*_{a2}(1-\cos\theta)]$
Actual chordal depth	$\bar{h}_p$	$\bar{h}_{p1}=\bar{h}_1+\frac{1}{2}(d'_{a1}-d_{a1})$	$\bar{h}_{p2}=\bar{h}_2+\frac{1}{2}(d'_{a2}-d_{a2})$

Note: The calculation equations for measuring teeth number of common normal line K, length of common normal line w_k, inclined diameter of dedendum circle Li and wave length of spiral line wave amplitude, please refer to Table 23.3-7.

5. Selection of basic parameters of circular arc gear drive

The basic parameters of circular arc gear drive: m_n, z, β, ε_β, φ_d and φ_a etc. have great affection to the bearing capability of drive and working quality, they have close relationship and mutual restrict between each parameter. The basic relationship between them should pay attention when selection:

$$d_1=z_1m_n/\cos\beta \qquad (4)$$

$$\varepsilon_\beta=b/p_x=b\sin\beta/\pi m_n \qquad (5)$$

$$\varphi_d=b/d_1=\pi\varepsilon_\beta/z_1\tan\beta=0.5\varphi_a(1+\mu) \qquad (6)$$

$$\varphi_a=b/a=2\varphi_d/(1+\mu)=2\pi\varepsilon_\beta/(z_1+z_2)\tan\beta \qquad (7)$$

While design, the comprehensive consideration shall be given as per the concrete conditions.

5.1 Tooth number Z and module m_n

when the center distance and tooth width of gear have been defined, take more teeth and reduce module accordingly, this is not only to increase coincidence degree and enhance the drive stability, but also to reduce relative sliding speed to improve drive efficiency and prevent from gluing. But if the module is too small, the bending strength of tooth will be not enough. So, under the condition of meeting bending strength of tooth, it should be appropriate to select smaller module.

Generally, to select $m_n=(0.01\sim0.02)$ a (a is center distance). For the large center distance, stable load and continuous working drive, select smaller value; and for the small center distance, unstable load and intermittent work drive, select larger value. In the general gear reducer, it's used to select $m_n=(0.0133\sim0.016)$ a. If it's special, for example the herring bone gear seat of rolling machine with outstanding peak load, may select $m_n=(0.025\sim0.04)$ a. If it's high speed drive, select smaller normal module for stable working.

In addition in the design, we may select the teeth number at first then determine the module. Generally, take $z_1\geq18\sim30$. Surface hardness $HB\leq350$, if load is not heavy, should select larger value; the surface hardness $HB>350$, the load is heavy, should select smaller value; If the speed is high select larger value. No undercut for circular arc gear, the min. teeth number shall not be restricted by undercut; but if the teeth number is less, the module is large, it's not easy to guarantee the value of coincidence degree.

5.2 Coincidence degree ε_β

To select larger coincidence degree may enhance the stability of drive, lower noise and improve bearing capability. For middle and low speed drive, we used to select $\varepsilon_\beta>2$; for high speed gear drive, we recommend $\varepsilon_\beta>3$ or larger value. When we adopt large coincidence degree, the tolerances of tooth spacing, tooth direction, axial parallelism and shafting deformation value should be restricted strictly, otherwise it could not guarantee that several contact tracks to bear load evenly and could not reach drive stably and proper bearing capability.

The coincidence degree consists of integer part μ_ε and mantissa $\triangle\varepsilon$, i.e. $\varepsilon_\beta=\mu_\varepsilon+\triangle\varepsilon$. The selected value of mantissa $\triangle\varepsilon$ of coincidence degree will have great affection to the bearing load capability and stability. Generally, the scope of value selection for mantissa $\triangle\varepsilon$ is $0.15\sim0.35$.

If $\triangle\varepsilon$ selected is too small, at the time of the contact track enter or break away from tooth surface, it's easy to cause tooth crest collapsed and not good for stable drive. If $\triangle\varepsilon$ increased, the stress at tooth crest will be reduced, but if $\triangle\varepsilon$ will be increased to above 0.4, the stress will be reduced slowly; if $\triangle\varepsilon$ selected is too much the tooth width increased so that not to increase contact track numbers at each twinkling.

5.3 Spiral angle β

The spiral angle β has more affections to drive quality. The β increased will cause equivalent curvature radius reduced so that lower the tooth surface contact strength and bending strength at dedendum, additionally, it will increase axial force and reduce bearings life. But if β increased, this will make coincidence degree ε_β increased, if we will obtain: $\varepsilon_\beta=2.15\sim3.35$ or $\varepsilon_\beta=3.15\sim3.35$, the drive stability, vibration and noise will be improved and contact strength and bending strength will be improved also. So to select β reasonably according to specific situation. General recommendation for helical gear, $\beta=10°\sim20°$; for herringbone gear, $\beta=25°\sim35°$.

5.4 Tooth width coefficient φ_d, φ_a

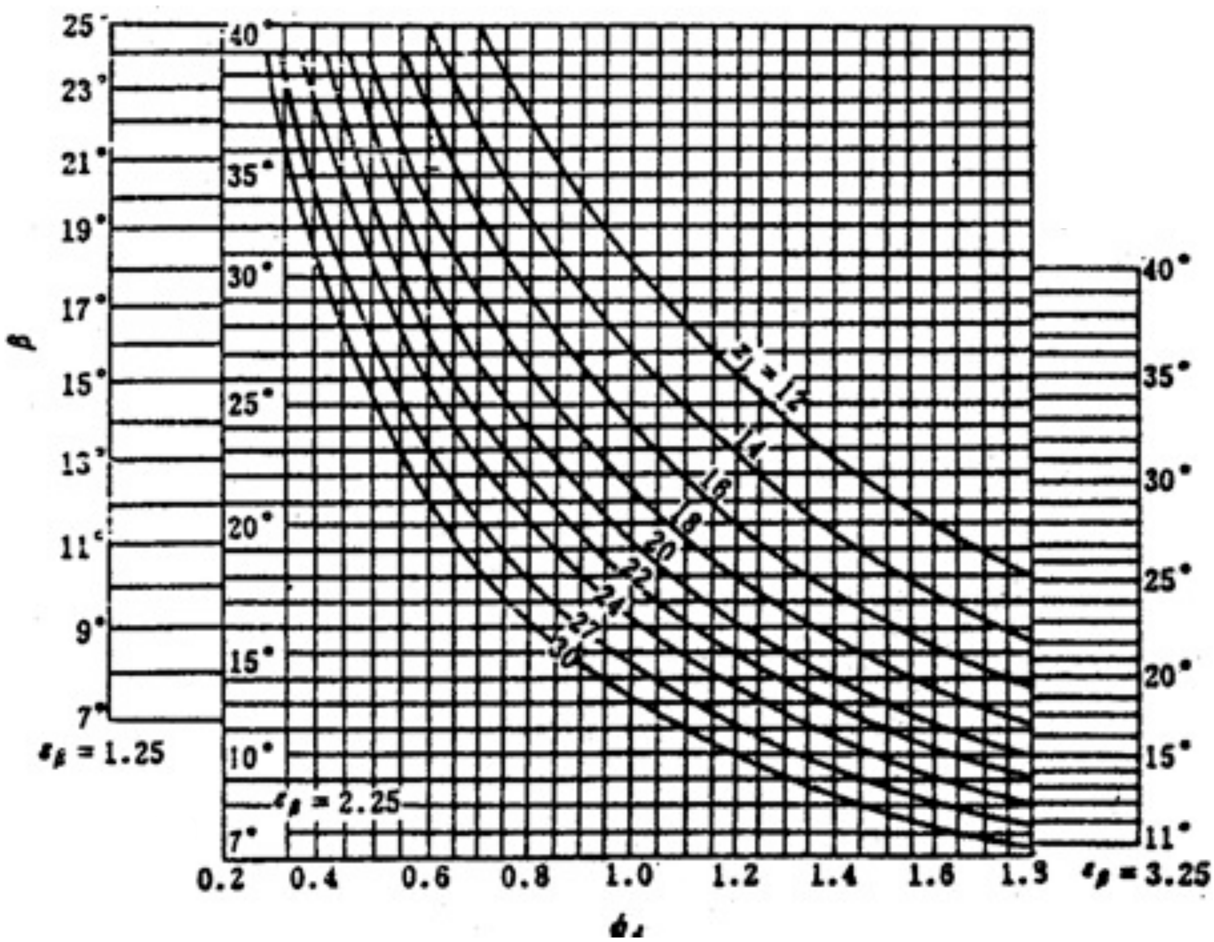

Fig. 7 Relationship between φ_d and z_1, β, ε_β

Tooth width coefficient $\varphi_d = \dfrac{b}{d_1}$, $\varphi_a = \dfrac{b}{a}$

Conversion relationship of φ_d and φ_a please refer to equations (6) and (7). When z_1, β and ε_β defined, check φ_d or φ_a according to equations (6) and (7). It's also to determine tooth width coefficient first and then to adjust z_1, β and ε_β's values with these equations.

When the values of ε_β are 1.25, 2.25, 3.25, use Fig.7 to select a group of suitable values of φ_d, z_1 and β.

6. Strength calculation of DCA gear

6.1 Strength calculation equation of DCA gear drive.

Please refer to Table 7 for the equation of bending strength of dedendum and strength of tooth surface contact for DCA gear drive.

Table 7 calculation equations of strength for DCA gear drive

Item	Calculation of bending fatigue strength of dedendum	Calculation of fatigue strength of tooth surface contact
Calculated stress N/mm²	$\sigma_F=\left(\frac{T_1K_AK_VK_1}{2\mu_\varepsilon+k_{\triangle\varepsilon}}\right)^{0.86}\frac{Y_EY_\mu Y_\beta Y_F}{z_1m_n^{2.58}}Y_{end}$	$\sigma_H=\left(\frac{T_1K_AK_VK_1K_{H2}}{2\mu_\varepsilon+k_{\triangle\varepsilon}}\right)^{0.73}\frac{Z_EZ_\mu Z_\beta Z_a}{z_1m_n^{2.19}}$
Normal module mm	$m_n\geq\left(\frac{T_1K_AK_VK_1}{2\mu_\varepsilon+k_{\triangle\varepsilon}}\right)^{1/3}\left(\frac{Y_EY_\mu Y_\beta Y_F}{z_1\sigma_{FP}}Y_{end}\right)^{1/2.58}$	$m_n\geq\left(\frac{T_1K_AK_VK_1K_{H2}}{2\mu_\varepsilon+k_{\triangle\varepsilon}}\right)^{1/3}\left(\frac{Z_EZ_\mu Z_\beta Z_a}{z_1\sigma_{HP}}\right)^{1/2.19}$
Torque of small gear N.mm	$T_1=\frac{2\mu_\varepsilon+k_{\triangle\varepsilon}}{K_AK_VK_1}\,m\ 3n\left(\frac{z_1\sigma_{FP}}{Y_EY_\mu Y_\beta Y_F\,Y_{end}}\right)^{1/0.86}$	$T_1=\frac{2\mu_\varepsilon+k_{\triangle\varepsilon}}{K_AK_VK_1K_{H2}}\,m^3_n\left(\frac{z_1\sigma_{HP}}{Z_EZ_\mu Z_\beta Z_a}\right)^{1/0.73}$
Allowance stress N/mm²	$\sigma_{FP}=\sigma_{FLim}Y_NY_X/S_{Fmin}\geq\sigma_F$	$\sigma_{HP}=\sigma_{HLim}Z_NZ_L/S_{Hmin}\geq\sigma_H$
Safety coefficient	$s_F=\sigma_{FLim}Y_NY_X/\sigma_F\geq S_{Fmin}$	$S_H=\sigma_{HLim}Z_NZ_L/\sigma_N\geq S_{Hmin}$

Note: For herringbone gear drive, the torque to be calculated according to $0.5T_1$, $(2\mu_\varepsilon+k_{\triangle\varepsilon})$ to be calculated as per half tooth width.

6.2 The signification of each parameter symbol and determination for each coefficient:

(1) Small gear tooth number Z_1, determined as per Chapter 5.1 of this artiale.

(2) The integer parts of superposition degree μ_ε, to reference this chapter 5.2

(3) Using coefficient K_A, to see chart 23.2-24. For high-speed gear drive, It should be recommended as per the experience to choose the 1.02—1.15 times chart value when v=40---70m/s; to choose 1.15---1.3 times chart value for v=70---100m/s, and to choose more than 1.3 times chart value for v›100m/s.

(4)dynamic load factor K_v, to reference to Fig 8.

（4）动载系数K_V，见图23·3-12。

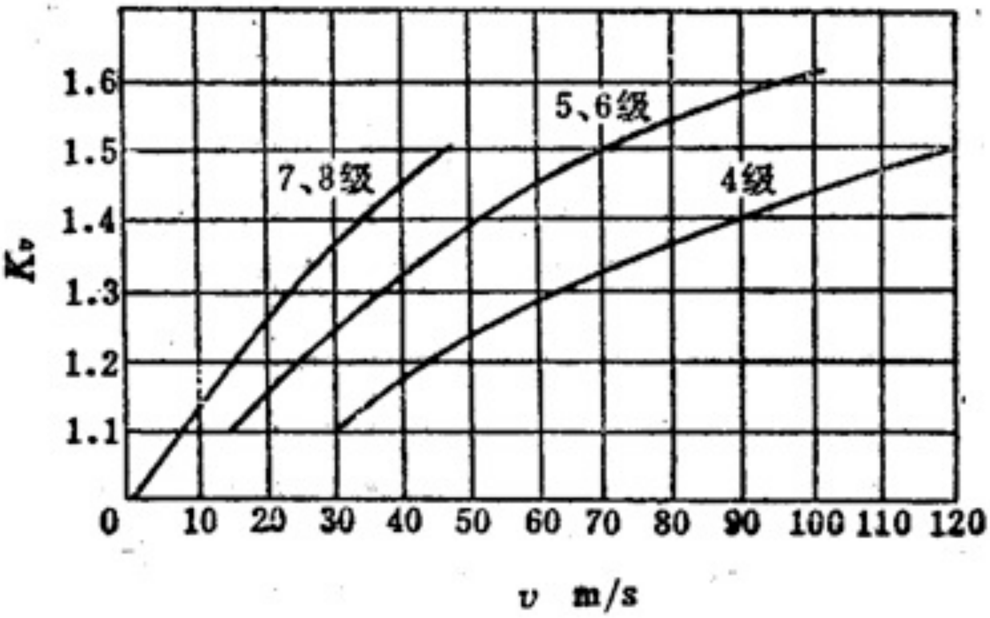

Fig. 8 Dynamic load factor K_V

(5) contact tracks loading assigning coefficient K_1, to reference to chart 23.3-13
(6) contact tracks loading assigning coefficient K_{F2}, K_{H2}, to reference to Table 8

Table 8 touching-mark loading assigning coefficient for contact tracks.

Grade		5	6	7	8
KF2		1			
KH2	81 Type	1.15	1.23	1.42	1.49

(7) contact tracks coefficient $K_{A\varepsilon}$, it is the coefficients considering that since superposition degree mantissa $\triangle\varepsilon$increase make the positive pressure decreasing for each contact tracks. The contact tracks coefficients of DCA gear drive are shown as Fig 9.

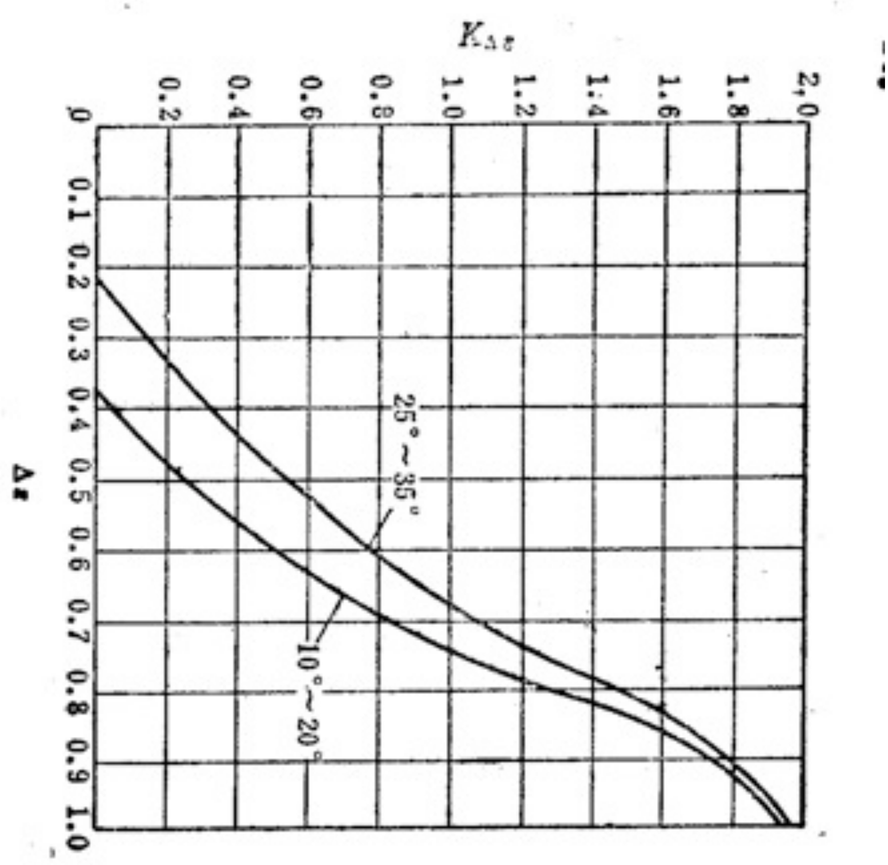

Fig 9 The contact tracks coefficient of DCA

(8) Elasticity coefficient Y_E, Z_E, To see Table 9

(9) gear number ratio coefficient Y_u, Z_u, to see Fig 10

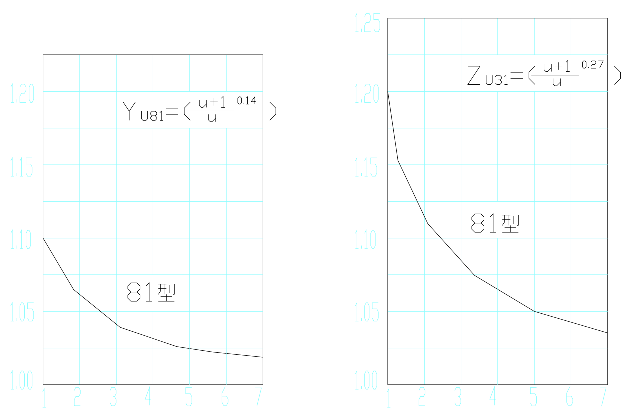

Fig 10 gear number ratio coefficient Y_u, Z_u

(10) spiral angle coefficient $Y\beta$, $Z\beta$, to see Fig 11

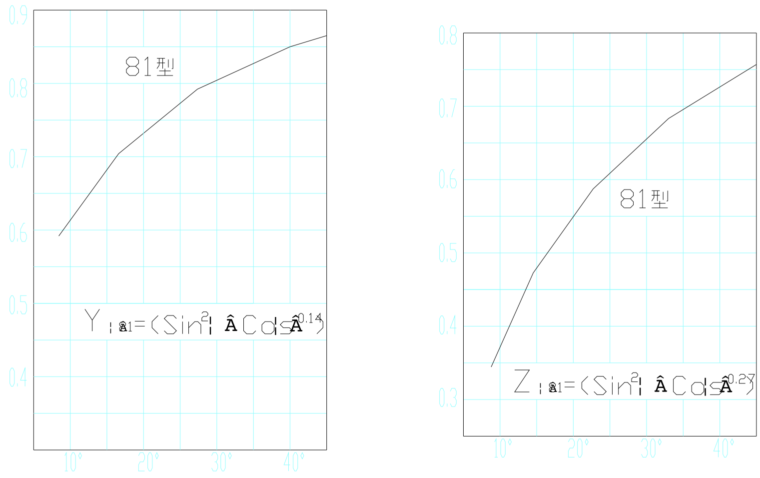

Fig 11 spiral angle coefficient Yβ， Zβ

(11) tooth form coefficient Y_F, to see Fig 12

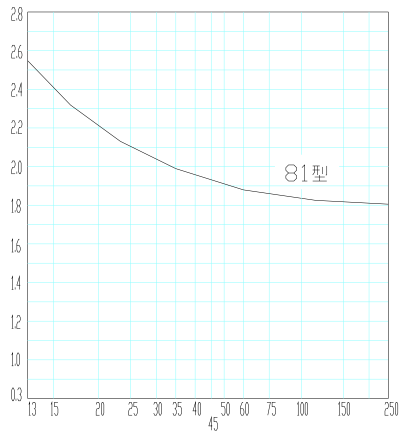

Fig 12 Tooth form coefficient Y_F

(12) tooth end coefficient Y_{end}, it is the coefficient considering the pressure of dedendum stress at tooth end will be increased when the instantaneously contact tracks is at tooth end. The value is the ratio of max. stress of end dedendum with max stress of the dedendum in the middle of tooth width, For tooth end coefficient of DCA gear, to see Fig 13, for gear which has been thin on the tooth end, the Y_{end}=1.

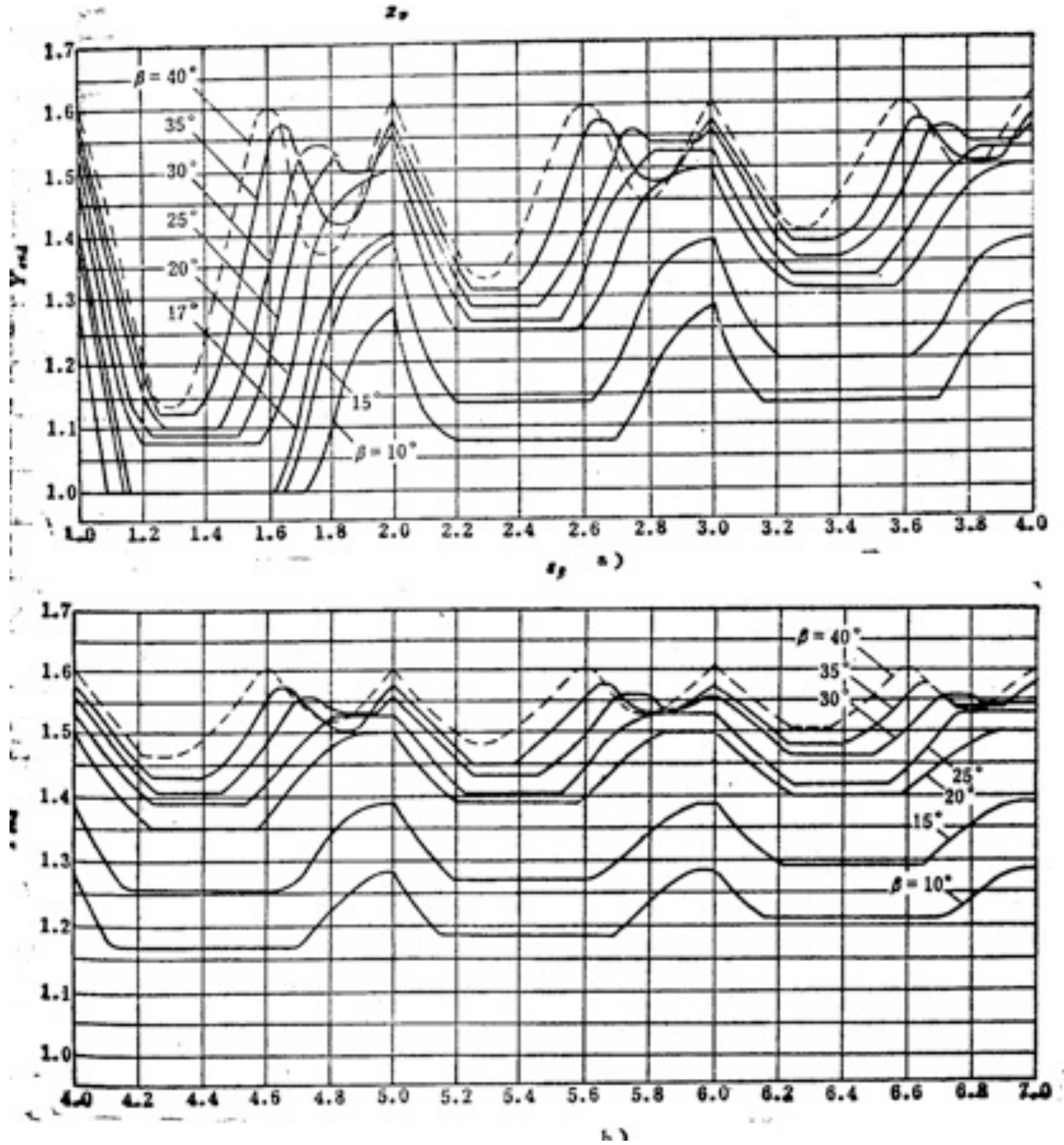

Fig 13

(13) contact-arc length coefficient Z_a, it's the coefficient considering the affections to contact arc length from module and equivalent teeth number. Please refer to Fig.14. for DAC gear, when the ratio of tooth number u is not 1, the up tooth surface and low tooth surface of a gear, their contact arc lengthes are not same, so the coefficient of contact arc length need to use the average value of Z_{a1} and Z_{a2}, i.e. $Z_{am} = 0.5 (Z_{a1} + Z_{a2})$.

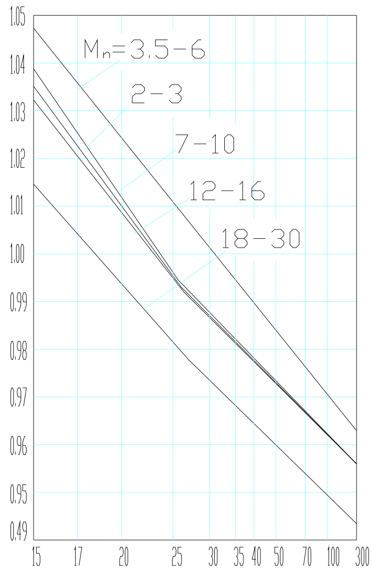

Fig 14 Contact arc length coefficient Z_a, For DAC gear $Z_{am}=0.5(Z_{a1}+Z_{a2})$

(14) the basic value of bending strength δ_{FE} for gear material to see Fig 15. it will be chosen the middle value of scope, the top value will be allowed to used when material is in good quality and better heat-treatment.

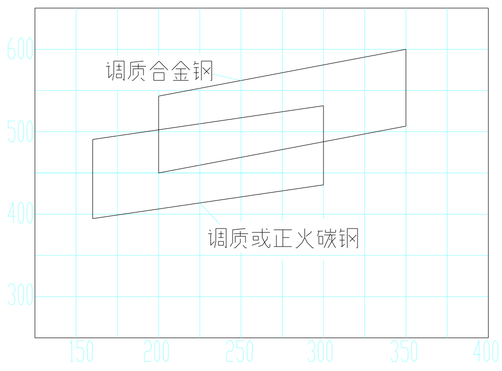

Fig 15

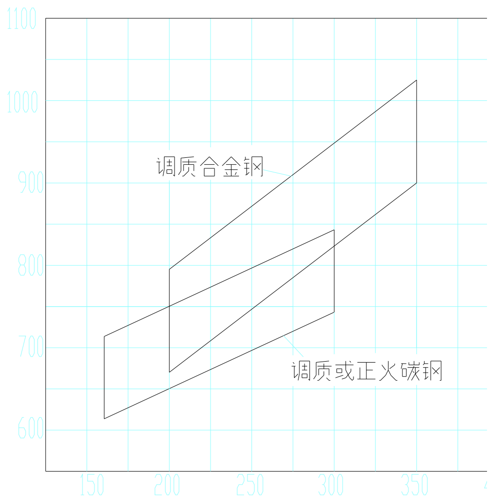

Fig 16

Table9 DCA elasticity coefficient Y_E, Z_E,

Tooth form	Symbol	Unit	A couple of forging gear	Other Material
DAC gear	Y_E	$(N/mm^2)^{0.14}$	2.073	$0.37E'^{0.14}$
DAC gear	Z_E	$(N/mm^2)^{0.27}$	31.37	$1.123E'^{0.27}$
E'=2/				

For gear working under the symmetry circulation stress, the value of δ_{FIlm} will be chosen as per the chart and it also will be 0.7times of chart value.

(15) Contact fatigue limitation stress δ_{HLim} of testing tooth surface, to see Fig 16.

Generally, it will be chosen the middle value of the scope. The top value of this chart will be allowed to choose when the material and is in good heat-treatment quality, and with the good structure to meet heat-treatment.

(16) life factor Y_N, Z_N,

(17) Size factor Y_X, to see Fig 17

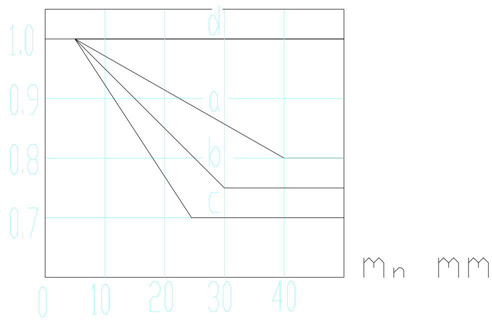

Fig 17 size factor Y_X of DAC gear

(18) lubricant factor Z_L,

(19) min safety factor S_{Fmin}, S_{Hmin}, to see table 10

Table10 The reference value of min safety factor

S_{Fmin}	1.6----1.8
S_{Hmin},	1.3----1.5

a- Quenched and tempered steel
b- Surface hardened steel
c- Casting steel
d- All material under the static load

When use of compound lubrication with less friction factor, for the value in Q+T steel gear, times factor 1.4; for the value in carbonized and quenched steel gear, times 1.1.

Ring Gear Radial Tolerance F_r

Precision Grade	Normal Module (mm)	Reference Circle Diameter (mm)					
		~125	>125 ~400	>400 ~800	>800 ~1600	>1600 ~2500	>2500 ~4000
5	2~3.5	14	16	18	-	-	-
	>3.5~6.3	16	18	20	22	-	-
	>6.3~10	20	22	22	25	28	-
	>10~16	22	25	28	28	32	36
	>16~25	-	32	36	36	40	40
	>25~40	-	-	45	45	50	50

6	2~3.5	22	25	28	-	-	-
	>3.5~6.3	28	32	32	36	-	-
	>6.3~10	32	36	36	40	45	-
	>10~16	36	40	45	45	50	56
	>16~25	-	50	56	56	63	63
	>25~40	-	-	71	71	80	80
7	2~3.5	36	40	45	-	-	-
	>3.5~6.3	45	50	50	56	-	-
	>6.3~10	50	56	56	63	71	-
	>10~16	56	63	71	71	80	90
	>16~25	-	80	90	90	100	100
	>25~40	-	-	112	112	125	125
8	2~3.5	50	56	63	-	-	-
	>3.5~6.3	63	71	71	80	-	-
	>6.3~10	71	80	80	90	100	-
	>10~16	80	90	100	100	112	125
	>16~25	-	112	125	125	140	140
	>25~40	-	-	160	160	180	180

Pitch Limiting Warp $\pm f_{pt}$ (μm)

Precision Grade	Normal Module (mm)	Reference Circle Diameter (mm)					
		~125	>125 ~400	>400 ~800	>800 ~1600	>1600 ~2500	>2500 ~4000
5	2~3.5	6	7	8	-	-	-
	>3.5~6.3	8	9	9	10	-	-
	>6.3~10	9	10	10	11	13	-
	>10~16	10	11	11	13	14	16
	>16~25	-	14	13	16	18	18
	>25~40	-	-	16	20	22	22
6	2~3.5	10	11	13	-	-	-
	>3.5~6.3	13	14	14	16	-	-
	>6.3~10	14	16	18	18	20	-
	>10~16	16	18	20	20	22	25
	>16~25	-	22	25	25	28	28
	>25~40	-	-	32	32	36	36
7	2~3.5	14	16	18	-	-	-
	>3.5~6.3	18	20	20	22	-	-
	>6.3~10	20	22	25	25	28	-
	>10~16	22	25	28	28	32	36
	>16~25	-	32	36	36	40	40
	>25~40	-	-	-	45	50	50

8	2~3.5	20	22	25	-	-	-
	>3.5~6.3	25	28	28	32	-	-
	>6.3~10	28	32	36	36	40	-
	>10~16	32	36	40	40	45	50
	>16~25	-	45	50	50	56	56
	>25~40	-	-	63	63	71	71

Axial pitch Internal Tolerance F_{β} (μm)

Precision Grade	Width of Gear (Axial Pitch) mm					
	~40	>40~100	>100~160	>160~250	>250~400	>400~630
5	7	10	12	16	18	22
6	9	12	16	19	24	28
7	11	16	20	24	28	34
8	18	25	32	38	45	55

Base Tangent Length Alteration Tolerance F_W (μm)

Precision Grade	Reference Circle Diameter (mm)					
	~125	>125~400	>400~800	>800~1600	>1600~2500	>2500~4000
5	12	16	20	25	28	40
6	20	25	32	40	45	63
7	28	36	45	56	71	90
8	40	50	63	80	100	125

Axial Parallel Tolerance

X axial Parallel Tolerance $f_x = F_{\beta}$	F_{β} see table above
Y axial Parallel Tolerance $f_x = \frac{1}{2} F_{\beta}$	

Center Limiting Warp $\pm f_a$ (μm)

Precision Grade	Center Distance (mm)

Grade	~120	>120 ~180	>180 ~250	>250 ~315	>315 ~400	>400 ~500	>500 ~630	>630 ~800	>800 ~1000	>1000 ~1250	>1250 ~1600	>1600 ~2000	>2000 ~2500	>2500 ~3150
5, 6	17.5	20	23	26	28.5	31.5	35	40	45	52	62	75	87	105
7, 8	27	31.5	36	40.5	44.5	48.5	55	62	70	82	97	115	140	165

Chordal tooth thickness Limiting Warp $\pm E_h$ (μm)

Precision Grade	Normal Module (mm)	Reference Circle Diameter (mm)										
		≤50	>50 ~80	>80 ~120	>120 ~200	>200 ~320	>320 ~500	>500 ~800	>800 ~1250	>1250 ~2000	>2000 ~3150	>3150 ~4000
5, 6	2~3.5	16	18	19	21	24	27	30				
	>3.5~6.3	20	21	23	25	27	30	34	37	41	45	50
	>6.3~10		25	27	30	32	34	37	41	45	50	60
7, 8	2~3.5	20	22	24	27	30	32	-	-	-	-	-
	>3.5~6.3	25	26	28	30	34	36	40	45	50	-	-
	>6.3~10	-	32	34	36	40	42	45	50	55	60	65
	>10~16	-	-	42	45	48	50	55	60	65	70	75
	>16~32	-	-	-	65	70	75	75	80	90	90	100

Note: for DCA gear, the Chordal tooth thickness Limiting Warp should be ±0.75 E_h

Root Circle Diameter Warp $\pm E_{df}$ (μm)

Precision Grade	Normal Module (mm)	Reference Circle Diameter (mm)										
		≤50	>50 ~80	>80 ~120	>120 ~200	>200 ~320	>320 ~500	>500 ~800	>800 ~1250	>1250 ~2000	>2000 ~3150	>3150 ~4000
5, 6	2~3.5	25	28	31	36	34	45	52	-	-	-	-
	>3.5~6.3	31	34	37	42	48	52	60	67	-	-	-
	>6.3~10	-	45	48	52	56	63	67	75	80	100	-
7, 8	2~3.5	30	34	38	44	50	55	-	-	-	-	-
	>3.5~6.3	40	44	48	50	55	66	70	80	-	-	-
	>6.3~10	-	55	60	65	70	75	80	90	100	-	-
	>10~16	-	-	75	80	85	90	100	110	120	140	160
	>16~32	-	-	-	120	125	130	140	150	160	180	200

Note: for DCA gear, the tolerance of Root circle diameter should be $\pm 0.75E_{df}$

Tooth Radicel Tolerance

Gear Precision Grade ①		5	6	7	8
Hole	Dim. Torlerance	IT5	IT6	IT7	
Shaft	Dim. Torlerance	IT5		IT6	
Tip Diameter ②		IT6	IT7		

Note: IT – unit of standard tolerance.

1 The tolerance is adopted as per the highest precision grade when precision grade of three groups tolerance are differents.

2 The tolerance is adopted as per IT11 when the tip diameter is not use of benchmark for teeth thickness and teeth depth, but not more than $0.1m_n$

Tooth Radicel Datum Plane Radial and Transverse Plane Tolerance (μm)

Reference Circle Diameter (mm)		Precision Grade	
From	To	5 and 6	7 and 8
--	125	11	18
125	400	14	22
400	800	20	32
800	1600	28	45
1600	2500	40	63
2500	4000	63	100

7.7 The formula for Limiting Warp and Tolerance

1) The value of tangent tolerance F_i', Tangent-teeth tolerance f_i', helix and spirals tolerance $f_{f\beta}$, Radial pitch Limiting Warp $\pm F_{px}$, X axial Parallel Tolerance f_x, Y axial Parallel Tolerance f_y, Center Limiting Warp $\pm f_a$ can be calculated as following formula:

$F_i' = F_p + f_\beta$
$f_i' = 0.6(f_{pt} + f_\beta)$
$f_{f\beta} = f_i' \cos\beta$
$f_{px} = f_\beta$
$F_{px} = F_\beta$
$f_x = F_\beta$
$f_y = 0.5\ F_\beta$
$f_a = 0.5$(IT6, IT7, IT8)

Here: β --- reference circle helix angle.

2) The value of base tangent length tolerance E_w and teeth thickness tolerance E_a can be calculated as following formula:
$E_w = -2\sin\alpha E_h$
$E_a = -2\tan\alpha E_h$
Here: α --- teeth angle.

3) The value of gear pair tangent tolerance $F_i'_c$ equal the sum of tolerance F_i' between two gear's tangent. When the ratio value of two teeth is a integer and not more than 3, F_i' can be expressed 25% or more than the calculated value.
The value of gear pair tangent – teeth tolerance $f_i'_c$ equal the sum of tolerance $f_i'_c$ between two gear's tangent – teeth.

4) The relationship between Limiting warp and tolerance and parameter of gear are listed as below:

Table: The relationship between Limiting warp and tolerance and parameter of the gear

Precision Grade	F_p	F_r	F_w	f_{pt}	$f\beta$	E_h	E_{df}

	$A\sqrt{L}+C$		$Am_n+B\sqrt{d}+C$ B=0.25A		$B\sqrt{d}+C$		$Am_n+B\sqrt{d}+C$ B=0.25A		$A\sqrt{b}+C$		$Am_n+B^{3}\sqrt{d}+C$			$Am_n+B^{3}\sqrt{d}$	
	A	C	A	C	B	C	A	C	A	C	A	B	C	A	B
4	1.0	2.5	0.56	7.1	0.34	5.4	0.25	3.15	0.63	3.15	0.96	1.92	2.88	1.92	3.84
5	1.6	4	0.90	11.2	0.54	8.7	0.40	5	0.80	4	1.2	2.4	3.6	2.4	4.8
6	2.5	6.3	1.40	18	0.87	14	0.63	8	1	5					
7	3.55	9	2.24	28	1.22	19.4	0.90	11.2	1.25	6.3	1.5	3	4.5	3	6
8	5	12.5	3.15	40	1.7	27	1.25	1.6	2	10					
Note	d- reference circle Dia.; b-face width; L- length of reference circle arc;														

Working Chart of DCA Gear (Drive sheave) Parts

Normal Module	m_n	3.5
Teeth Number	z	29
Teeth angle	α_n	24°
Dedendum	h_a*	0.9
Helix angle	β	15°44’26”
Direction of helix	Left	
The type of teeth	“81” type	
Teeth Depth	h	7
Nominal chordal depth	h	6.922
Precision Grade	8-8-7JB4021-85	
Gear pair center and limiting warp	$\alpha \pm f_a$	220 ± 0.036
mating gears	Drawing No.	
	Teeth Number	92
Tolerance Group	Inspection item code	Tolerance
Pitch tolerance	F_p	0.090
Pitch Limiting warp	f_{pt}	± 0.020
Radial pitch limiting warp	F_{px}	± 0.016

Chordal depth limiting warp	E_h	± 0.021
Actual chordal depth	h_x'= 6.922+ $\frac{1}{2}$ (d_a'-d_a)	

Technical specification:
1) The hardness should be 320-316HB after heat treatment.
2) Circle angle radius will be R2.5 if other radius not mentioned.

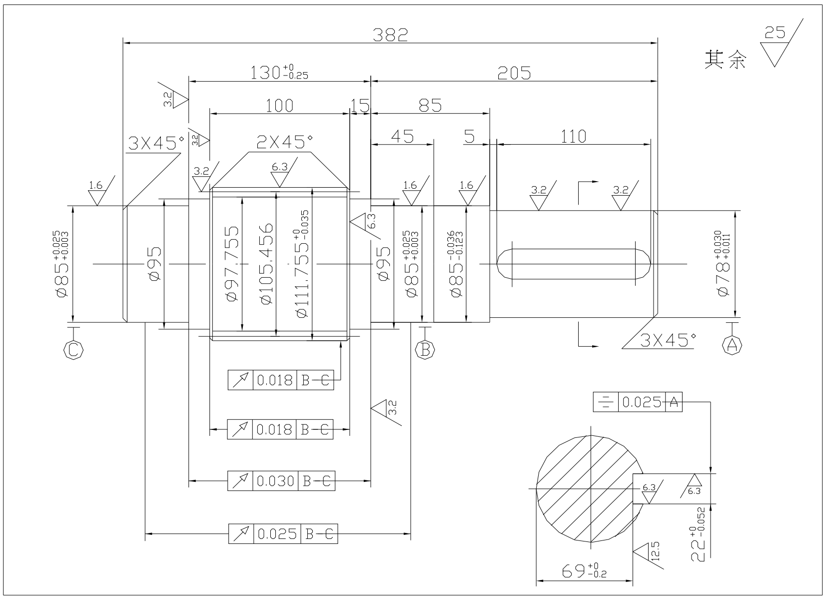

Fig. Working Sketch of DCA Gear (Drive sheave) Parts

Working Chart of DCA Gear (Driven sheave) Parts

Normal Module	m_n	3.5
Teeth Number	z	92
Teeth angle	α_n	24°
Dedendum	h_a*	0.9
Helix angle	β	15°44'26"
Direction of helix	Right	
The type of teeth	"81" type	
Teeth Depth	h	7
Nominal chordal depth	h	6.975
Precision Grade	8-8-7JB4021-85	
Gear pair center and limiting warp	α ± f_a	220 ± 0.036
mating gears	Drawing No.	
	Teeth Number	29
Tolerance Group	Inspection item code	Tolerance
Pitch tolerance	F_p	0.125
Pitch Limiting warp	f_{pt}	± 0.022
Radial pitch limiting warp	F_{px}	± 0.016
Chordal depth limiting warp	E_h	± 0.027

Actual chordal depth	$h_x' = 6.975 + \frac{1}{2}(d_a' - d_a)$

Technical specification:

1) The hardness should be 280-300HB after heat treatment.
2) The rough of tooth flank Ra should be 3.2μm.

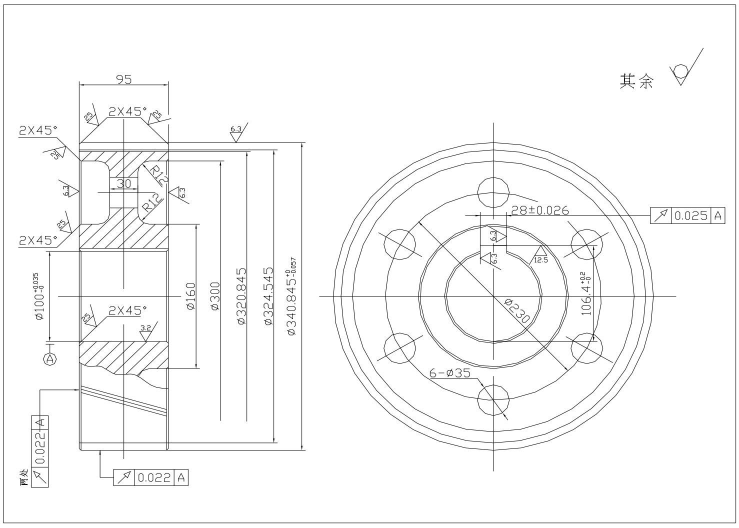

Fig Working Sketch of DCA Gear (Driven sheave) Parts

Please Note:

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